Statistics

Lecture 21

Feb 19-8:47 AM

SG 22 & SG 23

$\alpha \to$ Alpha

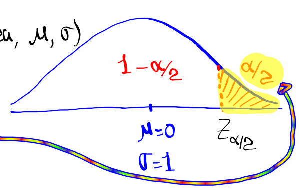
$\alpha \to$ Significance level

$0 < \alpha < 1$

$\alpha/2$ is the area of the right tail of the graph of Prob. dist.

$Z_{\alpha/2}$ is a number that separates the right area of $\alpha/2$ from the rest for standard normal Prob. dist. Always round to 3-decimal places.

We use invNorm(Left area, μ, σ) to find $Z_{\alpha/2}$.

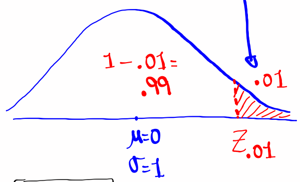


Nov 28-7:21 AM

Find $Z_{.01}$

$\frac{\alpha}{2} = .01$ Right Tail Area

$\alpha = .02$



$Z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

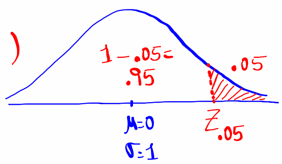
Find $Z_{\alpha/2}$ for $\alpha = .1$

$\alpha/2 = .1/2 = .05 \to$ Find $Z_{.05}$ Right-Tail Area

$\alpha/2$

$Z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$



Nov 28-7:26 AM

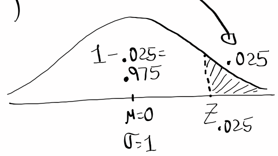
Find $Z_{\alpha/2}$ for $\alpha = .05$.

$\alpha/2 = .05/2 = .025 \to Z_{.025}$ Right-Tail Area

$\alpha/2$

$Z_{.025} = \text{invNorm}(.975, 0, 1)$

$= \boxed{1.960}$



Nov 28-7:33 AM

t - dist.

Graph is bell-shape, Symmetric with total area 1.

$\mu = 0$, σ is unknown

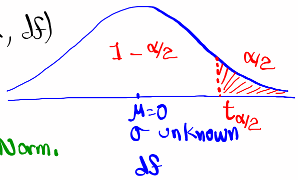
It comes with degrees of freedom df.

$t_{\alpha/2}$ is Similar to $z_{\alpha/2}$.

we use invT(Left Area, df)

$\boxed{2nd}$ $\boxed{VARS}$

It is usually below invNorm.



If You don't have it,
Download the Suggested Apps in the Syllabus.

Nov 28-7:37 AM

Find $t_{.02}$ with $df = 9$.

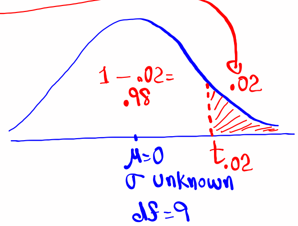
$\alpha/2 = .02$ Right-Tail Area

$\alpha = .04$

$t_{.02} = \text{invT}(.98 \,,\ 9\ )$

Left Area, df

$= \boxed{2.398}$



Find $t_{\alpha/2}$ for $\alpha = .06$ with $df = 14$.

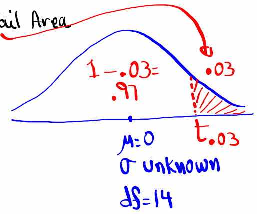
$\alpha/2 = .06/2 = .03 \rightarrow t_{.03}$

$\alpha/2$ Right-Tail Area

$t_{.03} = \text{invT}(.97, 14)$

Left Area, df

$= \boxed{2.046}$



Nov 28-7:43 AM

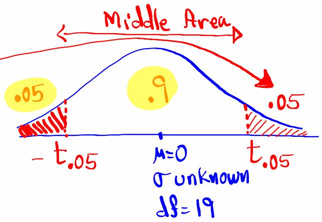
Find $\pm t_{\alpha/2}$ with df = 19 that separate the middle area of .9.

$1 - .9 = .1$

$.1 \div 2 = .05$

$\alpha/2$

Right Tail Area



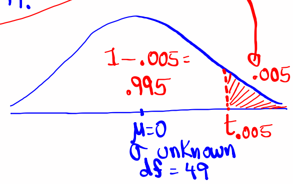
$t_{.05} = \text{invT}(.95, 19)$

$= \boxed{1.729}$

Find $t_{.005}$ with df = 49.

$\alpha/2 = .005$

$\alpha = .01$



$t_{.005} = \text{invT}(.995, 49)$

$= \boxed{2.680}$

Nov 28-7:50 AM

Estimating Parameters:

Statistic → Sample

Parameters → Population

We use statistic to estimate Parameters.

Estimation of any parameter will be in the form of range of values

Confidence Interval

Estimation of a parameter is in the form of Confidence Interval.

This process comes with some level of Confidence

C-level

Some Common C-levels are 90%, 95%, 99%.

Confidence level is the middle area of the graph of the Prob. dist. and is

$(1 - \alpha) \cdot 100\%$ where $0 < \alpha < 1$ and it is called Significance level.

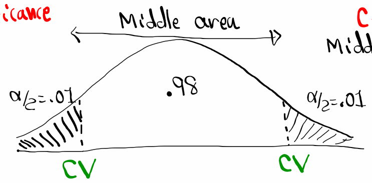
Nov 28-8:11 AM

If $\alpha = .02 \to (1-\alpha)\cdot 100\% = (1-.02)\cdot 100\% = 98\%$

Significance level

C-level

Middle area .98



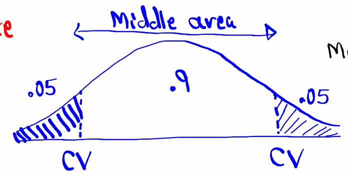
If $\alpha = .1 \Rightarrow (1-\alpha)\cdot 100\% = (1-.1)\cdot 100\% = 90\%$

Significance level

C-level

Middle area .9

$\alpha/2 = .1/2 = .05$



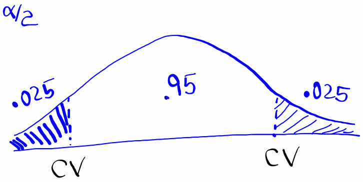
Nov 28-8:17 AM

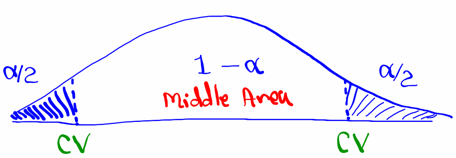
Do complete drawing for C-level = 95%

$1 - .95 = .05 \leftarrow \alpha$

$.05 \div 2 = .025 \leftarrow \alpha/2$

Middle area = .95





If α not given $\to$ Use .05

If C-level not given $\to$ Use 95%

Nov 28-8:22 AM

Estimating Population Proportion P:

Final Ans: Conf. Interval $\quad < p <$

$\hat{p} - E < p < \hat{p} + E$

P-hat
Sample Proportion
Point-estimate
" best guess obtained from Sample"

Margin of error

For example: $\hat{p} = .25$ & $E = .04$

$\hat{p} - E < p < \hat{p} + E$

$.25 - .04 < p < .25 + .04$

$\boxed{.21 < p < .29}$

Nov 28-8:28 AM

In a Sample of 100 Students, 75 of them had iphone.

$n = 100$
$x = 75$
$\Rightarrow \hat{p} = \frac{x}{n} = \frac{75}{100} = \boxed{.75}$

with margin of error of 5% $\rightarrow E = .05$

$\hat{p} - E < p < \hat{p} + E$

$.75 - .05 < p < .75 + .05$

$\boxed{.7 < p < .8}$

we estimate that between 70% & 80% of all Students have iPhone.

Nov 28-8:33 AM

In a Survey of 400 Students, 42% of them were in Support of Tuition-Free College.

$n = 400$ $\Rightarrow \hat{p} = \frac{x}{n}$ $\Rightarrow x = n\hat{p} = 400(.42) = \boxed{168}$

$\hat{p} = .42$

If decimal $\rightarrow$ Round-up

with 4% margin of error $\rightarrow E = .04$

$$\hat{p} - E < p < \hat{p} + E$$

$$.42 - .04 < p < .42 + .04$$

$$\boxed{.38 < p < .46}$$

We estimate that between 38% & 46% of all Students are in favor of tuition-free College.

Nov 28-8:37 AM

$$\hat{p} - E < p < \hat{p} + E$$

$\hat{p}$ Sample Proportion, Point-estimate, $\hat{p} = \frac{x}{n}$

$x \rightarrow$ # of favorable responces

$n \rightarrow$ Sample Size

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

$\hat{q} = 1 - \hat{p}$

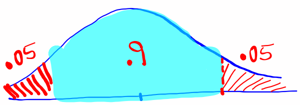
Critical value for $(1-\alpha)\cdot 100\%$ C-level.

Nov 28-8:47 AM

Suppose $n = 100$, $\hat{p} = .8$, C-level: 90%

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \cdot \sqrt{\frac{(.8)(.2)}{100}} = \boxed{.07}$

$x = n\hat{p} = 100(.8) = \boxed{80}$



$\mu = 0$
$\sigma = 1$

$Z_{.05} = \text{invNorm}(.95, 0, 1)$

$\hat{p} - E < p < \hat{p} + E$

$.8 - .07 < p < .8 + .07$

$\boxed{.73 < p < .87}$

we are 90% confident that Population Prop. falls between 73% & 87%.

Using TI Command:

STAT → TESTS → 1-PropZInt

$x = 80$
$n = 100$
C-level: .9
Calculate

$.734 < p < .866$

$\boxed{73\% < p < 87\%}$

$E = \frac{.866 - .734}{2} = \boxed{.07}$

$\hat{p} = \frac{.866 + .734}{2} = \boxed{.8}$

Nov 28-8:50 AM

In a survey of 185 students, 32% of them were in support of online classes.

1) How many of them were in support of online classes?

$n = 185$
$\hat{p} = .32$

$\rightarrow x = n\hat{p} = 185(.32) = 59.2 \rightarrow \boxed{x = 60}$

If decimal → Round-up

2) Construct Confidence interval for the prop. of all students in favor of online classes.

NO C-level
⇒ use .95

1-PropZInt

$x = 60$
$n = 185$
C-level: .95
Calculate

$.257 < p < .392$

$.26 < p < .39$

we are 95% confident that between 26% & 39% of all students are in support of online classes.

$E = \frac{.392 - .257}{2} = \boxed{.0675} \approx \boxed{.07}$

$\hat{p} = \frac{.392 + .257}{2} = \boxed{.325} \approx .33$

Nov 28-9:15 AM

I surveyed 580 voters and 135 of them were in support of certain item on the ballot.

$n = 580$

$x = 135$

$\rightarrow \hat{p} = \frac{x}{n} = \frac{135}{580} = .233 \rightarrow 23\%$

$\hat{q} = 1 - \hat{p} = .77 \rightarrow 77\%$

Construct $\boxed{99\%}$ Conf. interval for the prop. of all voters in support of that item.

$\rightarrow$ C-level: .99

$\boxed{.188 < p < .278}$

1-PropZInt

$x = 135$

$n = 580$

C-level = .99

Calculate

$.19 < p < .28$

$19\% < p < 28\%$

$E = \frac{.278 - .188}{2} = \boxed{.045} \approx \boxed{.05}$

$\hat{p} = \frac{.278 + .188}{2} = .233 \approx \boxed{.23}$

23%

we are 99% confident that between 19% & 28% of all voters are in support of that item.

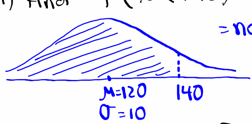
Nov 28-9:26 AM

Class QZ 11

Given $N(120, 10)$

Drawing, labeling, shading, and full TI-command needed.

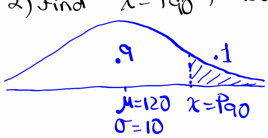
1) Find $P(x < 140)$



$= \text{normalcdf}(-E99, 140, 120, 10)$

$= \boxed{.977}$

2) Find $x = P_{90}$, Round to whole #.



$x = P_{90} = \text{invNorm}(.9, 120, 10)$

$= 132.816$

$\approx \boxed{133}$

Nov 28-9:37 AM